Case Based Question (CBQ) :

Case Study - 1

1. Students of a school are taken to a railway museum to learn about railways heritage and its history. `

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions.

   (i) Find whether the relation R is symmetric or not.

   (ii) Find whether the relation R is transitive or not.

   (iii) (a) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

   (iii) (b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

Case Study - 2

2. The government of India is planning to fix a hoarding at the face of a tower side by the road of a busy place for awareness on COVID-19 protocol.

Sanjeev, Rohit and Alok are three engineers who are working on this project. 'P' is considered to be a person viewing the hoarding 30 metres away from the tower, standing at the edge of a pathway nearby Sanjeev, Rohit and Alok suggested to the firm to place the hoarding at three different locationsnamely X, Y and Z. 'X' is at the height of 15 metres from the ground level. For the viewer P, the angle of elevation of 'Y' is double the angle of elevation of X. The angle of elevation of 'Z' is triple the angle of elevattion of 'X' for the same viewer.

Based on the given information, solve the following questions.

   (i) Find the measure of $\angle XPQ$.

   (ii) Find the measure of $\angle XP'Q$.

   (iii) (a) Find the measure of $\angle YPQ$.

OR

   (iii) (b) Find the measure of $\angle ZPQ$.

3. Amit, Biraj and Chirag were given the task of creating a square matrix of order 2.

Below are he matrices created by them. A, B, C are the matrices created by Amit, Biraj and Chirag respectively.

$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

If a = 4 and b = –2 .

Based on the above information, answer the following questions.

  (i) Find the sum of the matrices A, B and C.

 (ii) Find tthe value of $(A^T)^T$.

(iii) (a) Find the value of $(bA)^T + (aB)^T$.

<div align="center">OR</div>

(iii) (b) Find the value of AC – BC .

4. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens. 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

  (i) Form the equations required to solve the problem of finding the price of each item and express it in the matrix form AX = B.

 (ii) Find |A| and confirm if it is possible to find A^{-1}.

(iii) (a) Find A^{-1}, if possible and write the formula to find X.

<div align="center">OR</div>

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix.

5. A potter made a mud vessel, where the shape of the pot is based on $f(x) = |x - 3| + |x - 2|$, where $f(x)$ represents the height of the pot.

On the basis of above information, answer the following questions.

  (i) when x > 4 What will be the height in terms of x ?

 (ii) Will the slope vary with x value ?

(iii) (a) What is a at x = 3 ?

<div align="center">OR</div>

(iii) (b) When the x value lies between (2, 3), then the function $f(x)$ is

6. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since, he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions.

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in erms of x and its volume (V), which is constant.

(ii) Find $\dfrac{dS}{dx}$.

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum.

<div align="center">OR</div>

(iii) (b) If surface area (S) is constant, the volume

$(V) = \dfrac{1}{4}(Sx - 2x^3)$, x being the wedge of base. Show that volume (V) is maximum for $x = \sqrt{\dfrac{S}{6}}$.

7. For a function f(x), if f (–x) = f (x), then f (x) is an even function and f (–x) = –f(x), then f(x) is an odd function. Again, we have

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx, & \text{if } f(x) \text{ is even} \\ \\ 0, & \text{if } f(x) \text{ is odd} \end{cases}$$

Based on the above information, answer the following questions.

(i) $f(x) = x^2 \sin x$ is even or odd ?

(ii) What is the value of $\int_{-\pi}^{\pi} f(x)dx$?

(iii) (a) If g(x) = x sin x, then find the value of $\int_{-\pi}^{\pi} x\sin x \; dx$

<div align="center">OR</div>

(iii) (b) What is the value of $\int_{-\pi/2}^{\pi/2} |\sin x| dx$

8. Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have no been administered the drops.

By the end of 2nd week, half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\dfrac{dy}{dx} = k\left(50 - y\right)$, where x denotes the number of weeks and y the number of children who have been given the drops.

On the basis of above information, answer the following questions.

(i) State the order of the above given differential equation.

(ii) Which method of solving a differential equation can be used to solve $\dfrac{dy}{dx} = k\left(50 - y\right)$?

(iii) (a) Find the particular solution of the differential equation $\dfrac{dy}{dx} = k\left(50 - y\right)$ given that y(0) = 0 and k = 0.049 ?

<div align="center">OR</div>

(iii) Find the particular solution of the differential equation $\dfrac{dy}{dx} = k\left(50 - y\right)$ given that Y(I) = 49 and k = 0.049.

9. Consider the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}, \; \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$.

On the basis of above information, answer the following questions.

(i) Find the values of $\overrightarrow{BC}$ and $\overrightarrow{CA}$.

(ii) Find he area of ΔABC.

10. An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \dfrac{x-2}{3} = \dfrac{y+1}{-2} = \dfrac{z-3}{4}$, while the track for line B is represented by $l_2 : \dfrac{x-1}{2} = \dfrac{y-3}{1} = \dfrac{z+2}{-3}$

Based on the above information, answer the following questions.

(i) Find whether the two metro tracks are parallel.

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to bne positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$.

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point $(3, 2, 1)$. Determine the equation of the pedestrian walkway.

OR

(iii) (b) Find the shortest distance between Line A and Line B.

11. A shop selling electronic items sells smartphones of only hree reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

CBSE 2025

A person buys a smartphone from this shop.

(i) Find the probability that it was defective.

(ii) What is the probability that this defective smartphone was manufactured by company B ?

12. A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. the design includes a partition running parallel to one of the sides dividing the area (roof) into two section.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

Based on this information, answer the following questiions.

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.

(ii) Write the area of the solar panel as a function of x.

(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material. Considering the parallel partition.

13. A small town Is analyzing the pattern of a new street light installaion.

The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following

  (i) Differentiate the function $f(x)$

 (ii) (a) Find the intervals on which the $f(x)$ is increasing, $x \in [0, \pi]$.

<div align="center">OR</div>

 (ii) (b) Find the intervals on which $f(x)$ is decreasing, $x \in [0, \pi]$.

(iii) (a) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or loacl minimum or a point of inflexion.

14. Some students are having a misconception while comparing decimals. For example, a student may mention that 78.56 > 78.9 as 7856 > 789. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distance to which they have thrown the jevelin are shown below in the table.

Name of students	Distance of javelin (in meters)
Ajay	47.7
Bijoy	47.07
Kartik	43.09
Dinesh	43.9
Devesh	45.2

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions.

  (i) What is the probability of a student not having misconception but still answers Bijoy in the test ?

 (ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ?

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception ?

<div align="center">OR</div>

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ?

15. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike, the probability of many construction workers not being present for the job is 0.65.

The probability that many are not present and still the work gets compleed on time is 0.35. the probability that work will be completed on time when all workers are present is 0.80.

Let E_1 : represents the event when many workers were not presen for the job;

E_2 : represens the event when all workers were present; and

E : represents completing the constructioin work on time.

Based on the above information, answer the following questions :

  (i) What is the probability that all the workers are present for the job ?

 (ii) What is the probability that construction will be completed on time ?

(iii) (a) What is the probability that many workers are not present given thatt the construction work is completed on time ?

<div align="center">OR</div>

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ?

A N S W E R

1. (i) symmetric

     (ii) transitive

     (iii) (a) $y = 3x + c$, $c \in R$

     Or, (b) symmetric but not transitive.

2. (i) $\tan^{-1}\frac{1}{2}$

     (ii) $\tan^{-1}\frac{3}{8}$

     (iii) (a) $\tan^{-1}\frac{4}{3}$

     Or, (b) $\tan^{-1}\frac{11}{2}$

3. (i) $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

     (ii) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

     (iii) (a) $\begin{bmatrix} 14 & 6 \\ -4 & 14 \end{bmatrix}$

     Or, (b) $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$

4. (i) $\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

     (ii) $|A| = 50 \neq 0 \Rightarrow A^{-1}$ exists

     (iii) (a) $X = A^{-1} B$, $A^{-1} = \begin{bmatrix} 0 & -\frac{1}{10} & \frac{1}{5} \\ \frac{3}{5} & 0 & -2/_5 \\ -2/_5 & 1/_5 & 1/_5 \end{bmatrix}$

     Or, (b) $A^2 - 8I = \begin{bmatrix} 26 & 28 & 32 \\ 52 & 26 & 46 \\ 46 & 32 & 25 \end{bmatrix}$

5. (i) When $x > 4$, $f(x) = 2x - 5$

(ii) Yes $f'(x) = \begin{cases} -2 & , & x < 2 \\ 0 & , & 2 < x < 3 \\ 2 & , & x > 3 \end{cases}$

(iii) (a) $f'(3^-) = 0$, $f'(3^+) = 2$

Or, $\therefore f(x)$ is not differentiable at $n = 3$.

(b) $f(x) = 1$

6. (i) $S = 2x^2 + \dfrac{4c}{x}$

(ii) $\dfrac{ds}{dx} = 4x - \dfrac{4c}{x^2}$

(iii) (a) $x = y$

Or, (b) $\dfrac{d^2v}{dx^2}\bigg|_{x = \sqrt{\frac{5}{6}}} < 0$

7. (i) odd

(ii) 0

(iii) (a) 2π

Or, (b) 2

8. (i) 1

(ii) variable separable

(iii) (a) $0.049x + \log^{\frac{1}{50}}$

Or, (b) $0.049 x - 0.049$

9. (i) $\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

(ii) $\dfrac{\sqrt{210}}{2}$ sq unit

10. (i) not parallel

(ii) $\dfrac{x - 1}{3} = \dfrac{y + 2}{-2} = \dfrac{z + 3}{4}$

(iii) (a) $\dfrac{x - 3}{2} = \dfrac{y - 2}{17} = \dfrac{z - 1}{7}$

Or, (b) $\dfrac{31}{\sqrt{342}}$ units

11. (i) 0.0345

(ii) 0.4057

12. (i) $2x + 3y = 300$

(ii) $x\left(100 - \dfrac{2}{3}x\right)$

(iii) (a) $x = 75$; $A_{max} = 3750$ sq. unit

Or, (b) $y = 50$, $A_{max} = 3750$ sq. unit

13. (i) $e^x (\cos x + \sin x)$

(ii) (a) $\left[0, \dfrac{3\pi}{4}\right]$

Or (ii) (b) $\left[\dfrac{3\pi}{4}, \pi\right]$

(iii) $\dfrac{3\pi}{4}$

14. (i) 0.06

(ii) 0.38

(iii) (a) $\dfrac{16}{19}$

Or, (b) $\dfrac{3}{19}$

15. (i) 0.35

(ii) 0.51

(iii) (a) 0.45

Or, (b) 0.55